Absolute Maxima and Minima Word Problems

Warm-up

Given $f(x) = ax^2 + bx + c$. Find the critical points.

Summary of Absolute Extrema

> **If f is continuous on a finite closed interval, then the absolute extrema of f are guaranteed to exist and must occur at an endpoint or a critical point.**

> **If f is a continuous function on an open interval, then it *may* have absolute extrema. If it does have an absolute extremum, then it must occur at a critical point.**

Steps for Solving Maximum and Minimum Word Problems

1. _____

2. _____

3. _____

4. _____

Example 1 A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?

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Practice Problem 1 A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed using 1000 feet of fence.

Example 2 An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?
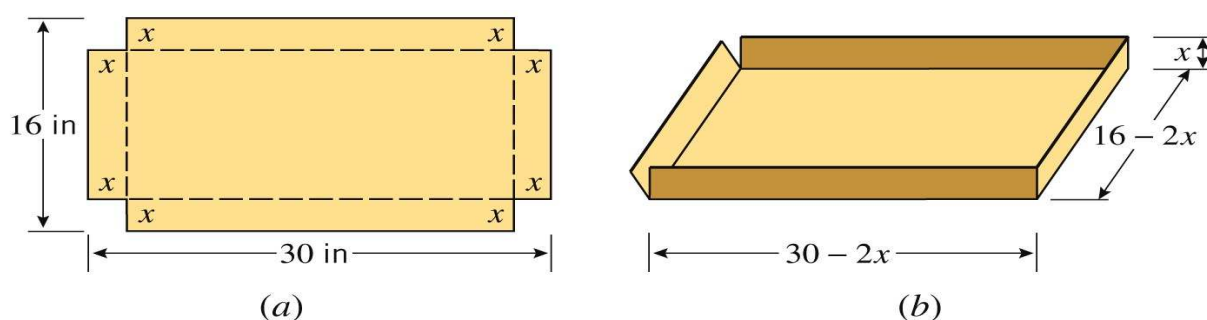
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Practice Problem 2 An open box is to be made from a 20-inch by 32-inch piece of cardboard by cutting out x-inch by x-inch squares from the four corners and bending up the sides. What size should the squares be to obtain the largest possible volume?

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Class Work

1. A positive number x and its reciprocal are added together. What is the smallest possible sum?

    a) Give the function to be maximized/minimized (in terms of x).

    b) Give the appropriate interval for x.

    c) Find the derivative.

    d) Find the critical points (eliminate those not in the interval.)

    e) Confirm that the critical point is the absolute max/min.

    f) Give the absolute max/min.

2. Two nonnegative numbers, x and y, have a sum equal to 10. What is the largest possible product of the two numbers?

    a) Give the function to be maximized/minimized (in terms of x).

    b) Give the appropriate interval for x.

    c) Find the derivative.

    d) Find the critical points (eliminate those not in the interval.)

    e) Confirm that the critical point is the absolute max/min.

    f) Give the absolute max/min.

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3. Find a number in the closed interval $\left[\frac{1}{2}, \frac{3}{2}\right]$ such that the sum of the number and its reciprocal is

    a) as small as possible?   b) as large as possible?

4. How should two nonnegative numbers be chosen so that their sum is 1 and the sum of their squares is

    a) as large as possible?   b) as small as possible?

5. The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences along its other two sides. Find the dimensions of the field with maximum area that can be enclosed using 1000 feet of fence.

6. A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for $3 a foot, while the remaining two sides will use standard fencing selling for $2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of $6000?